# Converting between reference ellipsoid datums

Latitiude and longitude coordinates in older Egyptian maps are referenced to the 1906 Helmert reference ellipsoid and are not compatible with the current WGS84 standard. To transpose between the two coordinate reference systems a transformation method is required. The following documentation is based on the calculations shown in the following document

*ERDC/TEC SR-00-1*
*USA Army corps of Engineers*

*Handbook for Transformation of Datums, Projections, Grids, and Common Coordinate Systems*

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## Reference ellipsoids

The Helmert ellipsoid has the following defining values :

$a_{06}$ = 6378200
$b_{06}$ = 6356818.16963
$rpf_{06}$ = 298.3
$f = (a - b) / a =$ 0.00335232986925921
$e_{06}^2 = (a_{06}^2 - b_{06}^2)/a_{06}^2 =$ 0.00669342162296610
$E_{06}^2 = (a_{06}^2 - b_{06}^2)/b_{06}^2 =$ 0.00673852541468365

The WGS84 ellipsoid has :

$a_{84}$ = 6378137
$b_{84}$ = 6356752.3142
$rpf_{84}$ = 298.257223563
$f = (a - b) / a =$ 0.00335281066474751
$e_{84}^2 = (a_{06}^2 - b_{06}^2)/a_{06}^2 =$ 0.00669437999014132
$E_{84}^2 = (a_{06}^2 - b_{06}^2)/b_{06}^2 =$ 0.00673949674227643

**1) Convert from latitude (φ) and longitude (λ) and altitude (h) to Cartesian :**

$X_{06}$ = ($R_N$ + h) cos φ cos λ
$Y_{06}$ = ($R_N$ + h) cos φ sin λ
$Z_{06}$ = ($b_{06}^2/a_{06}^2$) RN + h sin φ

where

$R_N = a_{06}/\mathrm{sqrt}(1-e_{06}^2.\sin^2\varphi)$

**2) Apply the 3 parameter transformation constants for the Helmert reference ellipsoid**

$X_{84}=X_{06}-130$
$Y_{84}=Y_{06}+110$
$Z_{84}=Z_{06}-13$

**3) Calculate longitude in WGS84 coordinates**

$\lambda=\arctan(X_{84}/Y_{84})$

**4) Calculate latitude in WGS84 coordinates**

The following iterative functions allow calculation of the latitude:

$\beta=\arctan (a_{84}\ Z)/(b_{84}.\mathrm{sqrt}(X_{84}^2+Y_{84}^2))$

$\varphi=\arctan (Z + E_{84}^2\ b \sin^3\beta\ )/(\mathrm{sqrt}(X^2 + Y^2) - a\ e_{84}^2 \cos^3\beta)$

$\beta=\arctan (1 - f) \tan \varphi$

5) **Calculate the altitude**

$h = \mathrm{sqrt}(X2 + Y2)/\cos \varphi - R_N$

$h = (Z/\sin \varphi) - R_N + e_{84}^2\ R_N$ for locations near the poles